Sperm motility algorithm: a novel metaheuristic approach for global optimisation

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Abstract: This paper proposes a new metaheuristic approach, namely, sperm motility algorithm (SMA), inspired by the fertilisation process in humans. Sperms are randomly diffused inside the female vagina to start searching for ovum. Investigation considering the modelling process of the sperm flow typical movement is carried out leading to selection of Stokes equations as mathematical model. A heuristic mechanism of sperms guided by chemoattractant secreted by ovum is to guarantee the progressing towards the goal. When the chemoattractant concentration increases the sperms are more likely to approach the ovum. Through the mimicking of the whole fertilisation process, a search approach to find a global optimisation algorithm is achieved. The proposed algorithm is tested using several standard benchmark functions as well as two engineering problems. A comparative study of the results with those obtained using well-known swarm intelligence algorithms is to validate and verify the efficiency of SMA. Getting the benefit of fertilisation chemoattractant, the proposed algorithm managed to solve unbounded constraint optimisation problems. A global optimal solution was reached in the solution of all benchmark problems proving the capability of the new algorithm to escape from local optimum.

Keywords: sperm motility; swarm intelligence; metaheuristic; optimisation.

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1 Introduction

Optimisation problems have been an important field and active area of research for several decades. The increasing complexities of life lead to complex optimisation problems; therefore, continuous development and improvement to optimisation algorithms is demanded in order to confront and resolve these problems. There are two general approaches to solve optimisation problems, namely, mathematical programming and metaheuristic methods (Kaveh and Talatahari, 2010). Mathematical programming as deterministic methods uses gradient information to search the solution space near an initial starting point. This is a gradient-based method where higher accuracy investigation fulfil local search task. However, the variables and cost function of the generators need to be continuous. Moreover, a good starting point is vital for these methods to be executed successfully. High computational effort is another drawback of deterministic gradient methods especially at high-dimensional search space. In many optimisation problems, prohibited zones, side limits, and non-smooth or non-convex cost functions need to be considered. As a result, these non-convex optimisation problems cannot be solved using traditional mathematical programming methods.

On the other hand, heuristic and metaheuristic methods rely on stochastic algorithms to generate different trade of solutions instead of gradients. However, the obtained solution values almost converge to same optimal solution with slight differences (Guo et al., 2014). Difference between heuristic and metaheuristic cannot easily be recognised (Yang, 2011). Heuristic means ‘to find’ or ‘to discover by trial and error’. In metaheuristic algorithms, meta – means ‘beyond’ or ‘higher level’, and they generally perform better than simple heuristics. All metaheuristic algorithms use certain trade-off of local search and global exploration.

Two important characteristics of metaheuristic are intensification and diversification (Gandomi et al., 2013). Intensification intends to search around the current best solutions and select the best candidates or solutions. Diversification makes sure that the algorithm can explore the search space more efficiently, often by randomisation. (Fister et al., 2013) presents brief review of nature-inspired metaheuristic algorithms, where all existing algorithms are divided into four major categories: swarm intelligence (SI)-based, bio-inspired (but not SI-based), physics/chemistry-based, and other algorithms. The SI is drawing inspiration from swarm-intelligence systems in nature depending on the collective behaviour of decentralised, self-organised systems, natural or artificial. Particle swarm optimisation (PSO) algorithm (Kennedy et al., 1995), ant colony optimisation (ACO) (Dorigo, 1992), bat algorithm (BA) (Yang, 2010a), cuckoo search (CS) (Yang and Deb, 2009), firefly algorithm (FA) (Yang, 2010b), krill herd (KH) (Gandomi and Alavi, 2012) are good examples for this category. The second category is bio-inspired (but not

SI-based): obviously, SI-based algorithms belong to a wider class of algorithms, called bio-inspired algorithms. In fact, bio-inspired algorithms form a majority of all nature-inspired algorithms. From the set theory point of view, SI-based algorithms are a subset of bio-inspired algorithms, while bio-inspired algorithms are a subset of nature-inspired algorithms. That is SI-based ⊂ bio-inspired ⊂ nature-inspired. Many bio-inspired algorithms do not use directly the swarming behaviour. Therefore, it is better to call them bio-inspired, but not SI-based. For example, genetic algorithms (GA), flower algorithm (FA) (Yang, 2012), differential evolution (DE) (Storn and Price, 1997), and human-inspired algorithm (Zhang et al., 2009).

The third category is physics and chemistry based: not all metaheuristic algorithms are bio-inspired because their sources of inspiration often come from physics and chemistry. For the algorithms that are not bio-inspired, most have been developed by mimicking certain physical and/or chemical laws, including electrical charges, gravity, river systems, etc., some good example for this category are harmony search (Geem et al., 2001), intelligent water drop (Shah-Hosseini, 2007), simulated annealing (Kirkpatrick, 1984) and stochastic diffusion search (Bishop, 1989). And the last category is other algorithms inspiration away from nature. Consequently, some algorithms are not bio-inspired or physics/chemistry-based, for example, differential search algorithm (Civicioglu, 2012), grammatical evolution (Ryan et al., 1998), and social emotional optimisation (Xu et al., 2010). Muller et al. (2002) developed optimisation algorithm based on bacterial chemotaxis, where the way in which bacteria react to chemoattractants in concentration gradients plays an important role in reaching the global optimal solution. Recently, a modified heuristic informed search techniques in ordered to improve the convergence speed and accuracy are introduced as: Mendes et al. (2004) presented the fully informed particle swarm FIPS, where considered that all neighbours particle are a source of influence no single best neighbour, this mean a particle is attracted to the best positions of all the particles. But in the traditional PSO algorithm, a particle is attracted toward the best position it has visited (with respect to an objective function). Zhang and Yi (2011) proposed scale-free fully informed PSO; they used a modified Barabási-Albert as a self-organising construction mechanism, in order to adaptively generate the population topology exhibiting scale-free property. They divided the swarm population into two subpopulations: the active particles and the inactive particles. The active particles fly around the solution space to find the global optima via iteratively updating their velocities and positions by using a novel weighted fully informed strategy; whereas the inactive particles are gradually activated by the active particles via attaching to them according to their own degrees, fitness values, and spatial positions. Qu et al. (2013) addressed a distance-based locally informed particle swarm model for multimodal optimisation. They used several local bests to guide the search of each particle instead of using the global best particle by using the information provided by its neighbourhoods. The neighbourhoods are estimated in terms of Euclidean distance. Oca and Stützle (2008) studied experimentally the convergence behaviour of the particles in FIPS when using topologies with different levels of connectivity. They showed that the particles tend to search a region whose size decreases as the connectivity of the population topology increases. Cushman (2007) presented a particle swarm approach to constrained optimisation informed by global worst. Where a global worst was determined for each iteration and added global worst term to velocity equation.

In this paper, a new algorithm inspired by the fertilisation process in human is proposed. Sperm spread in a random diffusion inside the female vagina to start searching for ovum. Investigation considering the modelling process of the sperm flow typical movement is carried out and lead to selection of ‘Stokes equations’ as mathematical model. A heuristic mechanism of sperms guided by chemoattractant secreted by ovum, is to guarantee the progressing towards the goal, when the chemoattractant concentration increases the sperms is more likely to approach the ovum. This algorithm includes additional measure to reach its goal through a chemoattractant concentration. This seems to be more efficient because it can work on the best sperms, rather than sequential sperms as in the other informed methods that depend on heuristic function measure between population (distance, degree, activity, direction, …, etc.).

The structure of this paper is organised as follows: Section 2 will introduce background on the human fertilisation process from kinetic point of view. Section 3, present the develop sperm motility algorithm (SMA). Section 4 present a set of well-known test functions along with two design engineering problems. The section also presents a performance-based comparison among CS, FA and PSO. Finally, conclusions are presented in Section 5.

2 Background

2.1 Preliminary

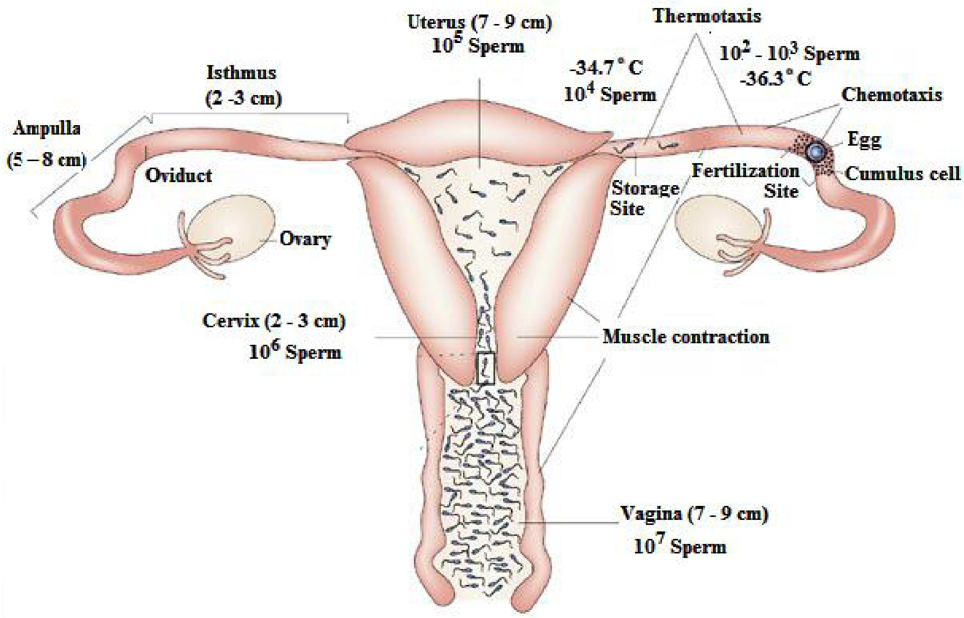
Sperm is the male reproductive cells, where reproduction requires the unification of male and female gametes. 1.5 to 5.0 ml of semen containing between 200 and 500 million sperm is randomly diffused at the posterior vaginal fornix (7–9 cm). After ejaculation begins one of the most important events in the fertilisation process is the sperm journey into the ovum. Sperm rapidly move towards the ovum. The first step, sperm moving from the vaginal to the cervix, where the endocervical canal (cervical canal) has an average length of 3.0 cm, and has several important functions as: filtering spermatozoa removal of seminal plasma, providing a biochemical environment sufficient for sperm storage, capacitation, and migration. Cervical mucus is continuously secreted serves many important functions, including exclusion of seminal plasma, exclusion of morphologically abnormal sperm, and support of viable sperm for subsequent migration to the uterus and oviduct. In the second step, moving about 10^5 of sperm into the uterus with a length of about 7–9 cm. More sperm lose their way for several reasons, the most important potentially hostile immune cells, so the overwhelming majority do not even reach the fallopian tubes, which has a length of about 7–9 cm. Sperm movement through the fallopian tube depends on the set of forces: intrinsic sperm motility, tubular muscular contraction, and fluid flow (Brannigan and Lipshultz, 2014). Tubal fluid production is maximal at the time of ovulation, and this fluid sustains the sperm before fertilisation. Tubal fluid may also facilitate both sperm capacitation and acrosomal reaction (Brannigan and Lipshultz, 2014). A few sperm arrive at the fertilisation site and one sperm penetrates the ovum, after that the ovum moves to the uterus. Sperm demonstration is shown in Figure 1.

2.2 Sperm guidance

Sperm movement refers to the ability of sperm to move into an ovum efficiently and properly. It was divided into four different grades according to World Health Organisation of classification in motility types (Imani et al., 2014):

- Type A Sperm with rapid progressive motility. They have the ability to reach the ovum and penetrate the membrane, and this type will be carried over to the next generations in our simulation.
- Type B Sperm with slow progressive motility and possibility of reaching the ovum but with less possibility of penetrating the membrane.
- Type C Non-progressive motility sperms. They do not move forward despite their different movement of tails.
- Type D Immotile or dead sperms.

Figure 1 Sperm demonstration (see online version for colours)



The effectiveness of sperm motility depends on several factors: in the vagina, cervix and uterus the sperm transport depends on sperm motility and muscular activity (Eisenbach and Giojalas, 2006). One of the most important factors in sperm guidance is chemoattractant concentration gradient by chemotaxis and thermotaxis, where chemotaxis, which is the movement of cells up a concentration gradient of chemoattractant, and thermotaxis which is the directed movement of cells along a temperature gradient (Eisenbach and Giojalas, 2006). May be first guided by thermotaxis from the cooler sperm storage site towards the warmer fertilisation site (Bahat et al., 2003). Sperm chemotaxis plays an important role in the process of fertilisation; the gradient of chemoattractant guides the sperm to the ovum (Alvarez et al., 2013). The ovum secretes a chemoattractant, which, as it spread away, forms a concentration gradient: a high concentration close to the egg, and a gradually lower concentration as the distance from the ovum is longer. Spermatozoa can sense this chemoattractant and orient their swimming direction up the concentration gradient towards the ovum. Chemotaxis is characterised by directional changes in the movement towards the source of the chemoattractant, and their swimming speed increases (this phenomenon is known as chemokinesis) (Eisenbach and Giojalas, 2006). There are three types of concentration, homogenous, gradient and point source. There are two types of gradient: spatial and temporal, Spatial gradient for the chemoattractant concentrations at different locations. And temporal gradient for the chemoattractant concentrations at different time points. The spatial gradient is a measurement of how the concentration of chemoattractant change from one source point to another, and can be described by this equations (1) and (2) (Friedrich and Jülicher, 2007; Lin et al., 2004):

- Linear concentration field:

  $$c(t) = c_0 + c_1 x(t) \tag{1}$$

  where c_0 = 10 pM (*pico-molar*) $c_1 = 0.1\left(c_0 / r_0 \right)$ where r_0 is a sperm moving length-scale

- Nonlinear spatial concentration gradient field:

  $$c(t) = c_0 + c_1 \left(x(t) \right)^b \tag{2}$$

  where $c(t)$ is the concentration, $x(t)$ is the position, c_1 and b are the proportion coefficient and the power of the major term position, respectively. c_0 represent the remaining terms.

2.3 Sperm motility mathematical framework

Many researchers studied the mathematical modelling of sperm motility as Taylor (1951), Hancock (1953), Gray and Hancock (1955), Pozrikidis and others (Montenegro-Johnson et al., 2012; Pozrikidis, 2002; Smith et al., 2009a, 2009b; Smith, 2009). Recently, two more approaches appeared to study sperm motility: The first approach by Elgeti (2006) and several researcher as Elgeti et al. (2010), Marx (2012), Yang (2009) and Yang et al. (2010) used molecular dynamics as multi-particle collision dynamics (MPC) also known as stochastic rotation dynamics (SRD) where in this simulation the time t is discrete. The second approach by Smith et al. (2009b) and Smith (2009) assumed that the fluid (sperm) flow obeying the Stokes equation (4). The stokes mathematical model is introduced in order to extract the analogy that will, later on, be used in the proposed algorithm to specify the particles movement in solution space.

$$\begin{gathered} \mathrm{Re}\left(\frac{\partial v}{\partial t} + v.\nabla v \right) + \nabla p = \mu \nabla^2 v + f, \\ \nabla \cdot v = 0, \quad x \in \Omega \end{gathered} \tag{3}$$

Leading in the limit Re → 0 to

$$\nabla p = \mu \nabla^2 v + f, \\ \nabla \cdot v = 0, \quad x \in \Omega \tag{4}$$

where p is the pressure, including the gravitational potential. v is the velocity vector field in the domain Ω, μ is kinematic viscosity and f is the force density. Reynolds number can be estimated as:

$$\text{Re} = \frac{\rho v^2 L^2}{\mu v L} = \frac{\text{inertial force}}{\text{viscous force}} < 10^{-2} \tag{5}$$

ρ is stress tensor, L is a characteristic length. Sperm swimming in low-Reynolds-number (10^{-2}) environment is good examples for swarm behaviour at small length scales (Yang, 2009).

The Stokeslet is the solution of the Stokes flow equations for unit force acting in the j-direction and concentrated at ζ, corresponding to taking

$$f(x) = \delta(x - \zeta) e_j \tag{6}$$

where δ is Dirac delta distribution centred at ζ, and e_j is the appropriate basis vector.

The i-component of the velocity field driven by this force is written as $S_{ij}(x, \zeta)$, and in an infinite fluid takes the form

$$S_{ij}(x, \zeta) = \left(\frac{\delta_{ij}}{r} + \frac{r_i r_j}{r^3} \right) \tag{7}$$

$S_{ij}(x, \zeta)$ is known as the Stokeslet, or Oseen-Burgers tensor, and

$$r_i = x_i - \zeta_i \text{ and } r^2 = | x - \delta |^2 = r_1^2 + r_2^2 + r_3^2 \tag{8}$$

δ_{ij} denotes the Kronecker tensor. The flow due to a force F concentrated at the point ζ corresponds to taking

$$f(x) = \delta(x - \zeta) F \tag{9}$$

The force density f in equation (4)

$$\vec{F} = \vec{F}_M + \vec{F}_N + \vec{F}_D + \vec{F}_C + \vec{F}_T \tag{10}$$

where elastic structure of the microtubules F_M, nexin/radial links F_N, dynein links F_D, forces that represent the cell wall (in the case of ciliary motion) or cell body (in the case of flagellar motion) F_C, and tethering forces that attach the axoneme to a cell wall or a cell body F_T.

The velocity solution corresponding to this fundamental singularity is given by (Smith, Gaffney, Blake, et al., 2009; Smith, 2009).

$$v_i(x) = (1/8\pi\mu) S_{ij}(x, \zeta) F_j \tag{11}$$

Smith et al. (2009b) and Smith (2009) updated the position using a technique based on the Heun second-order algorithm for the numerical solution of ordinary differential equations. In this work for simplicity proposed algorithm, we fixed orientation and we use trapezoidal second-order rule replace to Heun second-order algorithm for update position as the following:

$$\begin{aligned}&\textit{time step } n,\ \textit{position } x_0(t_n);\\&\textit{calculate velocity } v_n, v_{n+1} \textit{ from data at } t = t_n \textit{ and } t = t_{n+1}, \textit{respectively};\\&\textit{set } x_0(t_{n+1}) = x_0 + (\delta t/2)(v_n + v_{n+1})\\&\textit{go to time step } n+1.\end{aligned} \tag{12}$$

3 The proposed SMA

3.1 Proposed algorithm

For simplicity in describing SMA, we set several idealised rules which represent the rationale of SMA:

1. All sperm are attracted toward ovum of their species chemoattractant.
2. Attractiveness is proportional to chemoattractant concentration and those both increase whenever the sperm is close to ovum.
3. The best healthy or highest quality of sperm – type A – will be carried over to the next generations; other less quality sperms – types B, C and D – are neglect with a probability $P_a \in [0, 1]$.
4. One sperm penetrates the ovum, and this rule can be modified to suit the multi-objective optimisation as there more than egg as fraternal twins.
5. More than 250 million sperm swim randomly with velocity v_i at position x_i forward to ovum, where we can described the motility by the stokes equations.

Based on previous rules, the basic steps of the SMA can be summarised as the pseudo code shown below. Actually, sperm populations size N can be very large. However, in the computer simulations, we will use a far less population size. Each sperm is characterised by a position x_i and a continuous velocity v_i in the time t_i where $i = 1, 2, \ldots, N$. Calculation to generate the sperm position and velocity is performed using equations (7), (8) and (11). In equation (8), we assumed that position ζ is x_{i-1} then:

$$r_i = x_i - x_{i-1} \tag{13}$$

Assume g^* to be the term represented the most fit sperm. Using equation (12) to update the sperm position seeking for fertilisation by the fit sperm g^* yields the following update position equation:

$$x_{i+1} = x_i + (\delta t/2)(v_i + v_{i+1}) + \beta(x_i - g^*) \tag{14}$$

where g^* is the current best solution found among all solutions at the current generation/iteration. β is the number random to guarantee the diversity in the quality solutions. After that the chemoattractant concentration of equation (2) is checked. This value designates one of two cases: the first case is an active sperm or high quality sperm, that is moving towards the ovum, indicate that the distance decrease between the sperm and ovum, thus, concentration increases. The second case is an inactive sperm or worse sperm, which is not moving or moving away from ovum indicate that the distance increase between the sperm and ovum thus, the concentration will be either constant or decrease.

Sperm motility algorithm

Begin

Define objective function $f(x)$, $x = (x_1, x_1, \ldots, x_d)^T$

initialize N sperm population size

generate initial position x_0 *and velocity* v_0 *of N sperm*

define c_0, β, μ*…etc.*

while (*t < Maximum Generation*) *or* (*stoping criterion*);

  for *i=1: N do*

    calculate velocity v_i *from data at* $t = t_i$*; equation (11)*

    update position x_i *for sperm i from equation (14)*

    evaluate each sperm individual according to its position.

    if new solution is better, updated it in the population

    calculate c_i *from equation (2)*

    if $c_i \leq c_{i-1}$ *then neglect [Abandon a fraction* (P_a) *of worse sperm]*

    Check constraints satisfactions

  end for

Sort the population/sperm from best to worst and find the current best.

end while

Post-processing the results and visualization.

End

3.2 Constraint handling rules

Expansion and modification feasible-based mechanism proposed by Deb (2000) is used to handle the constraints problem and select the best individuals from one generation according to the following five rules:

Rule 1 Between two feasible sperms have same chemoattractant concentration the one with the higher fitness value is preferred.

Rule 2 Between two feasible sperms have same fitness value the one with the higher chemoattractant concentration is preferred.

Rule 3 Priority for chemoattractant concentration on fitness value in feasible solutions to avoid trapping into local optima.

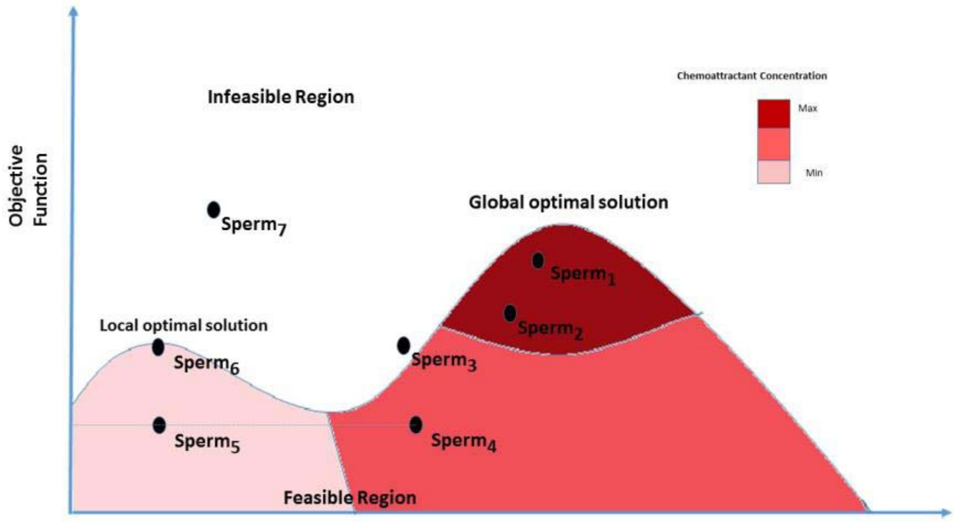
Rule 4 Any feasible sperm is preferred to any infeasible sperm.

Rule 5 If both sperms are infeasible, the one with the lowest sum of constraint violation is preferred. This sum is calculated as:

$$\sum_{i=1}^{n} \max\left(0, g(\vec{x})\right) + \sum_{j=1}^{p} \max\left(0, \left|h_j(\vec{x})\right| - \varepsilon\right) \tag{15}$$

Rule 1 guarantees that the search is directed to the feasible region at better solution. To get closer to the global optima, rule 2 is used. To avoid trapping into local optima, rule 3 is applied. Using rule 4, the search is oriented to the feasible region rather than to the infeasible region. To choose sperms leader, even when infeasible, lies closer to the feasible region, rule 5 is applied.

Figure 2 Graphical representation of constraint handling using the proposed algorithm (see online version for colours)



To understand this idea better, let us consider the following example:

- Let us consider sperms in Figure 2: comparing sperms 1 and 2, the two sperm have same chemoattractant concentration but different fitness value so sperm 1 wins. It has the highest fitness value.
- Comparing sperms 4 and 5, the two sperms have same fitness value but different chemoattractant concentration so sperm 4 wins since it has the highest chemoattractant concentration.
- Comparing sperms 4 and 6, the two sperms have different fitness value and different chemoattractant concentration so sperm 4 wins. It has the highest chemoattractant concentration although sperm 6 has highest fitness value.
- Comparing sperms 4 and 7, sperm 4 wins it is feasible.
- Comparing sperms 3 and 7, the two sperm are infeasible, according to rules 5 sperm 3 wins, it has smaller sum of constraint violation is preferred and closer to the feasible region.

4 Test benchmarked function

SMA is validated using some know benchmark problems obtained from Gandomi and Alavi (2012), Hezam et al. (2013), and Jamil and Yang (2013), and two design engineering problems (Gandomi et al., 2013; Raouf and Hezam, 2014). In addition, we will also compare the performance of the proposed algorithm with CS, FA, and PSO algorithms. The algorithms have been implemented by MATLAB R2011 on core (TM) i3, 2.27 GHz processor. Where the simulation parameter settings results of CS, FA, and PSO algorithms are as follows.

Table 1 Parameters of CS, FA, and PSO

CS	Number of nests n = 50, discovery rate of alien eggs/solutions p_a = 0. 25;
FA	Population size: 50, α (randomness): 0.25, minimum value of β: 0.20, γ (absorption): 1.0
PSO	Population size of 50, the inertia weight W, set to change from 0.9 ($w_{\max}$) to 0.4 ($w_{\min}$) over the iterations. Set weighting coefficients, c_1 = 0.12 and c_2 = 1.2.

Moreover, setting the parameters of the proposed algorithm of equations (2) and (11).

$\mu = 10^{-3}$ Pa/s is kinematic viscosity (Montenegro-Johnson et al., 2012) and F is the total force within the range (37–79 mW milliWatts) (Patrizio et al., 2000), β randomness scaling factor, this parameter is essential to guarantee the diversity of solutions. In equation (2) set c_0 = 10 pM (Friedrich and Jülicher, 2008), in this study we considered b is 0.5–2 (Lin et al., 2004).

4.1 Illustrative small scale problem

Sphere function (n = 2) have an exact minimum of 0 at (0, 0).

$$\min \sum_{i=1}^{2} x_i^2; -100 \le x_i \le 100; i = 1, 2.$$

4.2 Benchmark functions

The proposed SMA was tested using ten benchmark problems. The results are compared to three other swarms intelligent CS, FA and PSO. In order to investigate for the claimed improvements, the results obtained proved the capability of the proposed algorithm for reaching a competitive solution at a better optimised value. Referring to Table 4, it could be notice that a considerable improvement in the near optimal value was obtained compared to CS, FA, and PSO algorithms. The better optimised value could be referred to the chemoattractant formulation action that performs in a manner close to that of informed search techniques. The expected goal position at each iteration (best position) will attract all other agents (sperms) to make emphasised search in the nearby promising solution space. However, higher convergence time was obtained using SMA with respect to that of CS, FA, and PSO algorithms. Large mathematical computations at each iteration yield cumulatively a higher convergence time. More investigation is to be carried out later on to handle such deficiency.

Table 2 Different iterations for small scale problem using SMA algorithm

x_1	Initial SMA	−0.025479948191994	0.025854778770508	−0.002658259754476	−0.005855139436564
x_2		0.008659935267731	0.000746551883927	0.005640069975831	−0.002577322360900
$F(x)$		*7.242222387080e-004*	*6.690269249872e-004*	*3.887673425453e-005*	*4.092524837360e-005*
x_1	After 100 searches	−0.148290658966068	0.499875659515105	0.117573523090329	0.324140878085790
x_2		−0.280050723886956	−0.530196028370842	0.014757975611319	0.432926943459741
$F(x)$		*1.004185274862e-011*	*5.309835034758e-011*	*1.404133117602e-012*	*2.924930472196e-011*
x_1	After 300 searches	−0.023809149789759	0.480749622097440	−0.278650215682235	−0.315242082414503
x_2		0.214905334008240	0.383402147380209	0.183662107970459	−0.314940552882812
$F(x)$		*4.675117819890e-016*	*3.781174057625e-015*	*1.113777126039e-017*	*1.985651223752e-023*
x_1	After 500 searches	−0.158919719337045	0.913193087276705	−0.606142134404509	−0.118486771232835
x_2		0.291735724002266	0.310078576096799	0.490129565999616	0.127320742474475
$F(x)$		*1.103652098533e-037*	*9.300703380042e-063*	*6.076352785674e-107*	*3.024968642143e-124*

Table 3 The benchmark functions

ID	*Function name*	*Formulation*	*Global minimum*	*Range (bounds)*
F01	De-Jong function N:5 (*dimensions = 2*)	$\left(0.002+\sum_{i=1}^{25}\frac{1}{i+(x_1-a_{1i})^6+(x_2-a_{2i})^6}\right)$ $a=\begin{pmatrix}-32\ -16\ 0\ 16\ 32\ -32 \ldots 16\ 32\\ -32\ -32\ -32\ -32\ -16 \ldots 32\ 32\end{pmatrix}$	1	[–65.536, 65.536]
F02	Drop-wave function (*dimensions = 2*)	$\frac{1+\cos\left(12\sqrt{x_2^2+x_2^2}\right)}{2+0.5\left(x_1^2+x_2^2\right)}$	–1	[–5.12, 5.12]
F03	Hezam function (*dimensions = 2*)	$-\frac{1}{1+\lvert z^n+\tan z\rvert}$ $z\in C,\ n=20;\ z=x_1+ix_2\in[-2,2]$	1	$-2\le x_1, x_2, \le 2$
F04	Pathological function (*dimensions = 20*)	$\sum_{i=1}^{n-1}\frac{\sin^2\left(\sqrt{x_{i+1}^2+x_i^2}\right)-0.5}{0.001\left(x_i-x_{i+1}\right)^4+0.5}$	–1.9960079	$[-100, 100]^D$
F05	Powell function (*dimensions = 24*)	$\sum_{i=1}^{n/k}\left(x_{4i-3}+10x_{4i-2}\right)^2+5\left(x_{4i-1}+x_{4i}\right)^2$ $+\left(x_{4i-2}+x_{4i-1}\right)^4+10\left(x_{4i-3}+10x_{4i}\right)^4$	0	$[-4, 5]^D$
F06	Schaffer function N:4 (*dimensions = 2*)	$0.5+\frac{\cos\left(\sin\lvert x_i^2+x_2^2\rvert\right)-0.5}{\left(1+0.001\left(x_1^2 \_ x_2^2\right)\right)^2}$	0.292579	[–100, 100]
F07	Shekel function (*dimensions = 4*)	$\sum_{i=1}^{m}\frac{1}{c_i+\sum_{j=1}^{4}\left(x_j-a_{ji}\right)^2},\ m=10$	–10.5364	$[10, 10]^D$
F08	Sine Envelope function (*dimensions = 2*)	$-\sum_{i=1}^{n-1}\left[\frac{\sin^2\left(\sqrt{x_{i+1}^2+x_i^2}-0.5\right)}{\left(0.001\left(x_{i+1}^2+x_i^2\right)+1\right)^2}+0.5\right]$ $n=20$	0	$[-100, 100]^D$
F09	Styblinski-Tang function (*dimensions = 30*)	$\frac{\sum_{i=1}^{n}x_i^4-16x_i^2+5x_i}{2}$	$-39.16599D$	$[-5, 5]^D$
F10	Unbounded domains function (*dimensions = 3*)	$\left(1-x_1^2\right)^{0.5}\cos x_3+\frac{\left(1-x_2^2\right)^{0.5}}{\left(1+x_3^2\right)}+2x_3e^{-x_1}$	0	$-1\le x_1, x_2, \le 1,\ 0\le x_3\le\infty$

Table 4 Result comparisons on ten test functions

ID	*Function*		*SMA*	*CS*	*PSO*	*FA*
F01	De-Jong function N:5	Best	0.998003837	*0.998003838*	12.6705058	0.9980038378
		Mean	0.998003838	1.044446848	12.6705058	1.143360007
		Std. dev.	4.3E-010	1.1E-002	2.22E-011	3.3E-002
		Time (s)	80	96	19	136
F02	Drop-Wave function	Best	*–1.00E+00*	*–1.00E+00*	–0.9999995	–0.999999999
		Mean	–1.00E+00	–0.99999992	–0.99997	–0.99999997
		Std. dev.	0.00E+00	1.9E-007	3.02E-005	2.19E-008
		Time (s)	128	100	*23*	*118*
F03	Hezam function	Best	*–1*	–0.9901	–0.9955	–0.9950
		Mean	-0.9853	–0.9822	–0.9656	–0.9848
		Std. dev.	0.0090	0.0067	0.0208	0.0079
		Time (s)	25	20	22	18
F04	Pathological function	Best	*–53.24 E+00*	–41.215E+00	–40.45E-003	–47.2E-001
		Mean	–52.89E+00	–40.77E+00	–36.33E-004	–44.33E-001
		Std. dev.	7.06E-002	2.2E-001	7.21E-004	3.24E-001
		Time (s)	160	154	28	137
F05	Powell function	Best	*2.91E-016*	3.99E-013	1.74E-010	5.27E-013
		Mean	2.27E-016	2.44E-011	4.9E-008	2.98E-010
		Std. dev.	2.758E-018	3.47E-011	6.36E-008	5.40E-010
		Time (s)	109	112	61.2	83.3
F06	Schaffer function N:4	Best	*0.500E+00*	0.5000009	0.500E+00	0.500001
		Mean	0.500E+00	0.500037	0.500011	0.500044
		Std. dev.	0.00E+00	3.29E-005	2.82E-005	3.26E-005
		Time (s)	135	126	111	201
F07	Shekel function	Best	*–10.5364097*	–10.5364095	–10.53429	–10.53640
		Mean	–10.5364E+00	–10.536322	–10.530721	–10.5363872
		Std. dev.	0.00E-00	1.77E-004	4.08E-005	9.27E-002
		Time (s)	95	124	64	60
F08	Sine Envelope function	Best	*–0.749E+00*	0.00E+00	0.00E+00	0.00E+00
		Mean	–0.735E+00	1.41E-004	5.57E-004	3.78E-004
		Std. dev.	3.12E-004	2.54E-004	3.65E-004	3.99E-004
		Time (s)	6	5	3.5	8
F09	Styblinski-Tang function	Best	*–1.1749E+003*	–1.14E+003	–1.050E+003	–1.161E+003
		Mean	–1.169E+003	–1.1E+003	–9.82E+002	–1.085E+003
		Std. dev.	26.02E+002	37.66E+00	35.24E+002	45.68E+00
		Time (s)	66.9	63	5.4	59.9
F10	Unbounded domains	There exist four global minimisers of $f(x)$ at $x_1 = \pm 1$, $x_2 = \pm 1$, and $x_3 = 0$, where $f(x) = 0$.				

The proposed SMA algorithm proved the capability of handling special cases as in F03 where F03 is a multi-peak function problem in the complex plane. Locating multiple peaks simultaneously introduces extra difficulties for the existing optimisation methods. The function presents large peak centred at coordinates (0, 0) and surrounded by 19 thin peaks at height 1.0. Which reaches global minimum –1.

Drop-wave, Schaffer4, Pathological, and Sine Envelope's problems are complex multimodal functions with a large number of local optima. The basin of the global minimum is very narrow and global minimum value is different from the best local optimum. As the local optima are not punctual, they form crowns around the global optimum, there are in fact an infinite number of local optima that form a sort of a trap around the global optimum.

Therefore, difficult to reach unless a lucky start is made from within the domain of attraction of the global minimum. Concentration of chemoattractant may be exploited to help local search to escape from local minima and distribute its search efforts in the search space. However, SMA algorithm was capable of locating the global maximum of the function.

The proposed SMA algorithm proved the capability of handling unbounded domains as in F10 where an unbounded feasible solution is present (Ratschek and Voller, 1990). There exist four global minimisers of $f(x)$ at $x_1 = \pm 1$, $x_2 = \pm 1$, and $x_3 = 0$, where $f(x) = 0$; the SMA was reached for all solutions. The algorithm managed to attract the agents moving in open space to the area where the most promising solution is expected. Relevant obtained solution is not guaranteed using other SI algorithms at large solution spaces.

4.3 Industry engineering problems

4.3.1 Corrugated bulkhead design

Corrugated bulkhead design is often used in chemical tankers and product tankers in order to help facilities cargo tank washing effectively. This problem is as an example of minimum-weight design of the corrugated bulkheads for a tanker. Four design variables of the problem are width (b), depth (h), length (l), and plate thickness (t) for minimum-weight design of the corrugated bulkheads for a tanker, the mathematical formula for the optimisation problem as follows (Gandomi et al., 2013):

$$\text{Minimise}: f(b, h, l, t) = \frac{5.885t(b+l)}{b+\sqrt{(l^2-h^2)}}$$

$$g_1 = th\left(0.4b+\frac{1}{6}\right)-8.94\left(b+\sqrt{(l^2-h^2)}\right) \geq 0$$

$$g_2 = th^2\left(0.2b+\frac{1}{12}\right)-2.2\left(8.94+\left(b+\sqrt{(l^2-h^2)}\right)\right)^{\frac{4}{3}} \geq 0$$

$$g_3 = t-0.0156b-0.15 \geq 0$$

$$g_4 = t-0.0156l-0.15 \geq 0$$

$$g_5 = t - 1.05 \geq 0$$

$$g_6 = l - h \geq 0,\ 0 \leq b, h, l \leq 100 \text{ and } 0 \leq t \leq 5$$

The comparison result of minimum-weight and the statistical values of the best solution obtained by the SMA algorithm and Gandomi et al. (2013) are given in Table 5. The best minimum-weight in this study is 7.008391 with thickness 1.05 cm. while using Gandomi et al. (2013) obtained 5.894331, with thickness 0.7306255. In Gandomi et al. also the constraint g_5 is unverified and must be at least 1.05. Results are obtained from SMA algorithm is better than the results obtained using Gandomi et al. (2013). It gave us a greater thickness, with a slight increase in minimum-weight and verify all the constraints of given problem. While Gandomi et al. (2013) got minimum-weight but did not verify all the constraints. Thickness is very small leading of poor quality. If we exclude thickness constraint we could get a better result than that of (5.894331). Since t is between (0–5) when t approaches to zero, b also approaches to zero.

Table 5 Comparison results of the corrugated bulkhead design example

	b (cm)	*h (cm)*	*l (cm)*	*t (cm)*	*Best*	*Average*	*SD*
SMA	57.69231	37.26590	57.69231	1.05	7.008391	7.0093	0.0012
Gandomi et al. (2013)	37.117949	33.035021	37.193939476	0.73062	5.894331	5.988257	0.06436

4.3.2 Design of a gear train

Figure 3 shows the gear train problem (Gandomi et al., 2013; Raouf and Hezam, 2014). A gear ratio between the driver and driven shafts must be achieved when designing a compound gear train. The gear ratio for gear train is defined as the ratio of the angular velocity of the output shaft to that of the input shaft. It is desirable to produce a gear ratio as close as possible to 1/6.931. For each gear, the number of teeth must be between 12 and 60. The design variables T_a, T_b, T_d, and T_f are the numbers of teeth of the gears a, b, d and f, respectively, which must be integers.

$$\vec{x} = (T_d, T_b, T_a, T_f)^T = (x_1, x_2, x_3, x_4)^T$$

The optimisation problem is expressed as:

$$\min z = \left(\frac{1}{6.931} - \frac{\lfloor T_d \rfloor \lfloor T_b \rfloor}{\lfloor T_a \rfloor \lfloor T_f \rfloor}\right)^2 = \left(\frac{1}{6.931} - \frac{\lfloor x_1 \rfloor \lfloor x_2 \rfloor}{\lfloor x_3 \rfloor \lfloor x_4 \rfloor}\right)^2$$

$$\text{subject to } 12 \leq x_i \leq 60 \quad i = 1, 2, 3, 4$$

The constraint ensures that the error between obtained gear ratio and the desired gear ratio is not more than the 50% of the desired gear ratio.

Figure 3 A gear train (see online version for colours)

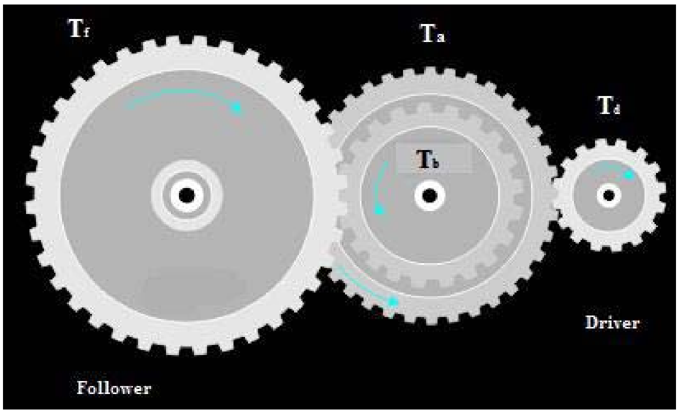


Figure 4 Memory usage indicator (see online version for colours)

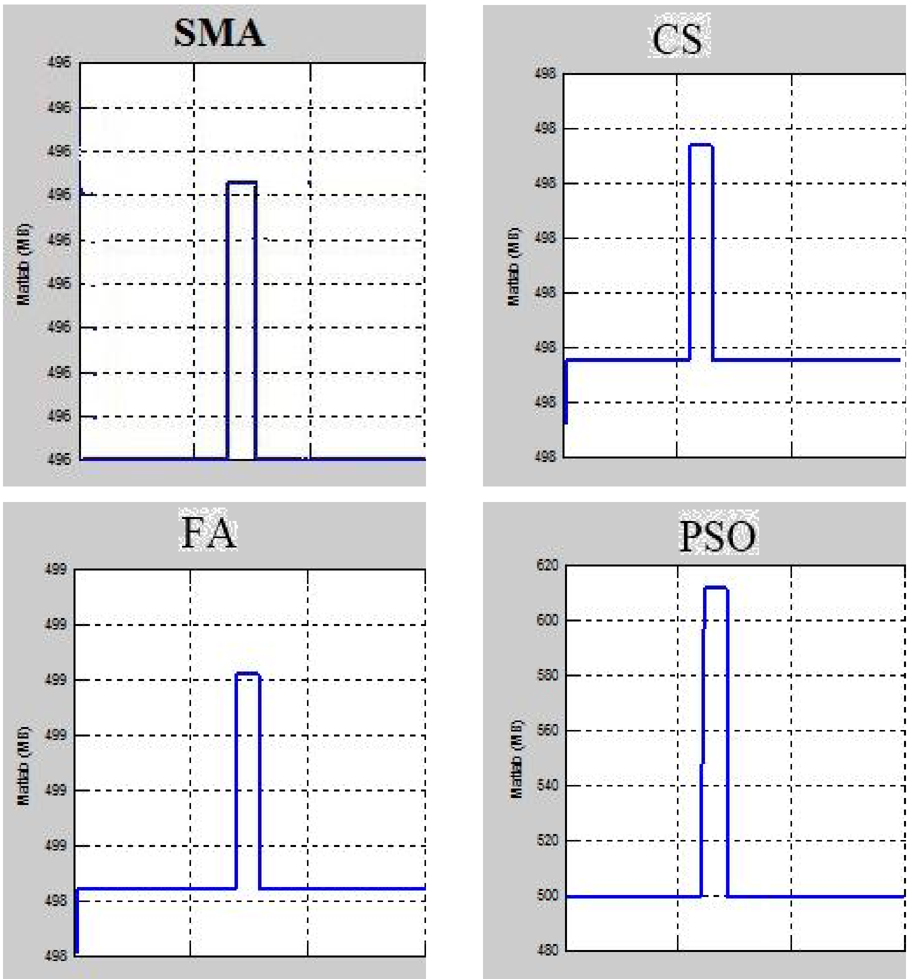


Table 6 Comparison results of the SMA, CS, FA and PSO

	SMA	*CS*	*FA*	*PSO*
Best	*2.7E-012*	2.70009E-012	*2.7E-012*	2.700857E-012
Error (%)	*0%*	9.0000e-017%	*0%*	8.5700e-016%
Mean	2.7E-012	1.04E-010	5.1314E-012	1.1371E-011
Std. dev.	0.00E+00	2.7E-010	7.6868E-012	1.01307E-011
Time (s)	95	65	65	*45*
Memory utilisation	*496*	498	499	600–620

The comparison resulted obtained by the SMA, CA, FA and PSO algorithms are given in Table 6, The comparison in terms of the best, error, mean, standard deviation values, convergence time and the amount addressed memory resources. These values where obtained out of 20 independent runs.

The result indicates a better achievement for SMA, except the convergence time. It could be referred to the large number of parameters and complex equations for velocity calculation. The proposed algorithm managed to obtain lower memory utilisation levels as shown in Figure 4.

5 Conclusions

A novel sperm motility metaheuristic algorithm, inspired by the fertilisation process in human was developed. Investigation considering the selection of the best mathematical modelling process of the sperm flow typical movement is carried out. The 'Stokes equations' was chosen as the best representing model to sperm flow. A heuristic mechanism of sperms guided by chemoattractant secreted by ovum was mathematically modelled making it more likely to the moving agent (sperm) to approach the goal (ovum). A search approach algorithm to find a global optimisation algorithm is achieved. The proposed algorithm is tested using several standard benchmark functions and two engineering problems. A comparative study of the results versus those obtained using well-known SI algorithms were introduced to validate and verify the efficiency of SMA. Getting benefit of chemoattractant, the proposed algorithm managed to solve unbounded constraint optimisation problems. A global optimal solution was reached regarding all the used benchmark problems, proving the capability of the new algorithm to escape from local optimal. The proposed algorithm can extend to handle multi-objective optimisation where more than one ovum as in fraternal twins could be assumed.

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